Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2012

Marking Scheme

Mathematics
(Project Maths – Phase 1)

Ordinary Level

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GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att (3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

APPLYING THE GUIDELINES

Examples (not exhaustive) of the different types of error:

Blunders (i.e. mathematical errors) (-3)

- Algebraic errors : $8x + 9x = 17x^2$ or $5p \times 4p = 20p$ or $(-3)^2 = 6$
- Sign error -3(-4) = -12
- Decimal errors
- Fraction error (incorrect fraction, inversion etc.)
- Cross-multiplication error
- Operation chosen is incorrect. (e.g., multiplication instead of division)
- Transposition error: e.g. $-2x - k + 3 \Rightarrow -2x = 3 + k$ or $-3x = 6 \Rightarrow x = 2$ or $4x = 12 \Rightarrow x = 8$;.
- Distribution error e.g. $3(2x+4)$ has $6x+4$ or $\frac{1}{2}(3-x) = 5 \Rightarrow 6 - x = 5$
- Omission, if not oversimplified.
- Index error.
- Factorisation: error in one or both factors of a quadratic: $2x^2 - 2x - 3 = (2x-1)(x+3)$
- Root errors from candidate's factors: error in one or both roots:
- Error(s) in transcribing formulae from tables (assuming it generates mathematical acceptable answer(s)) Serious errors or over simplifications will merit Attempt marks at most (check relevant section of scheme)
- Central sign error in *uv* or *u/v* formulae
- Omission of $\div v^2$ or division not done in *u/v* formula
- Vice-versa substitution in *uv* or *u/v* formulae
- Quadratic formula and its application apply a maximum of two blunders

Slips (-1)

- Numerical slips: 4 + 7 = 10 or 3×6 = 24, but 5 + 3 = 15 is a blunder.
- An omitted round-off or incorrect round off to a required degree of accuracy, or early rounding offs which effects final answer are penalised as a slip each time.
- However an early round-off which has the effect of simplifying the work is at least a blunder
- Omission of units of measurement or giving the incorrect units of measurement in an answer is treated as a slip, once per section of each question. Only applies where a candidate would otherwise have achieved full marks

Misreadings (-1)

- Writing 2436 for 2346 will not alter the nature of the question so M(-1)
  However, writing 5000 for 5026 will simplify the work and is penalised as at least a blunder.

Note: Correct relevant formula *isolated* and stops: if formula is *not* in Tables, award attempt mark.

QUESTION 1

Part (a)	**15 marks**	**Att 5**
Part (b)	**15 (5, 10) marks**	**Att (2, 2, 2)**
Part (c)	**20 (10, 10) marks**	**Att (2, 2, 3)**

* Incorrect or omitted units: penalise as per guidelines.

Part (a) **15 marks** **Att 5**

When Katie had travelled 140 km, she had completed $\frac{4}{9}$ of her journey.

Find the length of her journey.

(a) **15 marks** **Att 5**

I $\frac{4}{9}$ of journey = 140 km $\Rightarrow$ length of journey = $\frac{140\times 9}{4}$ (140×2.25) [12] = 315 km [15]

or

II $\frac{140}{4} = 35$ [9] $\Rightarrow$ length of journey = 35×9 [12] = 315 km [15]

or

III $\frac{4}{9} = 0\cdot 4444$ [9] $\Rightarrow$ length of journey = $\frac{140}{0.4444}$ [12] = 315.0315 km [15]

* Accept correct answers without work

Blunders (-3)

B1 Mathematical error e.g. decimal point (apply if method III does not give answer 315 to nearest whole number)

Slips (-1)

S1 Units omitted

Attempts (5 marks)

A1 Any relevant work

Worthless (0)

W1 Incorrect answer without work

W2 $140\div 9$ and stops

W3 Divides 140 in the ratio 4:9 and stops

Part (b) **15 (5, 10) marks** **Att (2, 2, 2)**

Robert's electricity bill gave the following data:

Unit type	*Present reading*	*Previous reading*	*Unit price*
Day rate	35 087	34 537	€0·1506
Night rate	17 213	16 853	€0·0745

(i) Calculate the total cost of the units used.
Robert also pays a standing charge of €24·89 and a levy of €5·46.
VAT at the rate of 13·5% is charged on all amounts.
(ii) Calculate the total amount of Robert's electricity bill.

(b) (i) **5 marks** **Att 2**

Day: 35087 – 34537 = 550 units. Cost: $550 \times 0.1506 = €82.83$

Night: 17213 – 16853 = 360 units. Cost: $360 \times 0.0745 = €26.82$
Total cost: €82.83 + €26.82 = €109.65.

Blunders (-3)
B1 Mathematical error

Slips (-1)
S1 Numerical slips

Attempts (2 marks)
A1 Any relevant work e.g. mentions/states cost = units × rate

(b) (ii) **10 (5, 5) marks** **Att (2, 2)**

Total:
€109.65 + €24.89 + €5.46 = €140 [5]

Amount with VAT
I Vat = $140 \times 0.135 = €18.90$ [4] ⇒ Total amount = €140 + €18.90 = €158.90 [5]
or
II Total amount = 140×1.135 [2] = €158.90 [5]

* Accept candidates work from (i)

Blunders(-3)
B1 Mathematical error

Slips (-1)
S1 Numerical slips

Misreading (-1)
M1 Omits standing charge or levy

Attempts (2 marks)
A1 Any relevant work

Part (c) **20 (10, 10) marks** **Att (2, 2, 3)**

A retailer bought 40 toys at €24·75 each.
He sold 10 of the toys at €33·88 each and sold the remaining 30 toys at a reduced price.
His total sales amounted to €1270.

(i) Write his total profit on the transaction as a percentage of his cost.
Give your answer correct to one decimal place.

(ii) Find the reduced selling price of each of the remaining 30 toys.

* Answers to parts of question must be clearly identified/labelled otherwise assume order in paper

(c) (i) **10 (5, 5) marks** **Att (2, 2)**

Cost: $40 \times €24.75 = €990$ [5]

Profit: $€1270 - €990 = €280$

Percentage profit: $\frac{280}{990} \times 100 = 28.28 = 28.3\%$

[2] [4] [5]

Blunders (-3)
B1 Mathematical error e.g. finding percentage

Slips (-1)
S1 Fails to round off or rounds-off incorrectly

Attempts (2marks)
A1 Any relevant work (two attempts possible) e.g. uses single unit costs.

Note: $\frac{1270}{990} = 1.2828$ and continues can merit full marks

(c) (ii) **10 marks** **Att 3**

Selling price, 10 toys: $10 \times €33.88 = €338.80$ [4]
Selling price, 30 toys: $€1270 - €338.80 = €931.20$ [7]
Price per toy: $€931.20 \div 30 = €31.04$ [10]

Blunders (-3)
B1 Mathematical error

Slips (-1)
S1 Numerical slips

Attempts (3 marks)
A1 Any relevant work

QUESTION 2

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (15, 5) marks**	**Att (5, 2)**
Part (c)	**15 (5, 10) marks**	**Att (2, 3)**

Part (a) **15 marks** **Att 5**

Solve for x and y

$$x - y = 4$$
$$2x + y = 5.$$

(a) **15 marks** **Att 5**

$$\begin{array}{l} x - y = 4 \\ \underline{2x + y = 5} \\ 3x \quad = 9 \quad \Rightarrow \quad x = 3 \qquad [12] \end{array}$$

$$x - y = 4 \quad \Rightarrow \quad 3 - y = 4 \quad \Rightarrow \quad -y = 1 \quad \Rightarrow \quad y = -1 \quad [15]$$

* Accept fully correct answer without work
* Accept one correct answer without work for 12 marks
* Candidates may offer other correct methods e.g. may eliminate x

Blunders (-3)

B1 Mathematical error once only in finding the first variable

Attempts (5 marks)

A1 Any relevant work e.g. may attempt to graph lines

Part (b) **20 (15, 5) marks** **Att (5, 2)**

Let $f(x) = x^3 + 2x^2 - x - 2$. **(i)** Show, by division, that $x - 1$ is a factor of $f(x)$. **(ii)** Hence, or otherwise, find the other factors of $f(x)$.

(b) (i) **15 marks** **Att 5**

$$
\begin{array}{r|l}
 & x^2 + 3x + 2 \\ \hline
x - 1 & x^3 + 2x^2 - x - 2 \\
 & \underline{x^3 - x^2} \\
 & \quad 3x^2 - x \\
 & \quad \underline{3x^2 - 3x} \\
 & \qquad 2x - 2 \\
 & \qquad \underline{2x - 2} \\
 & \qquad\quad 0
\end{array}
$$

	x^2	$+3x$	$+2$
x	x^3	$+3x^2$	$+2x$
-1	$-1x^2$	$-3x$	-2

Array Method of division.

* Accept synthetic division or other methods e.g. Array Method

Blunders (-3)
B1 Mathematical error
B2 Fails to show by division e.g. finds $f(1) = 0$

Misreading (-1)
M1 Uses $x + 1$ i.e $f(-1) = 0 \Rightarrow$ 11 marks (M (-1) and B(-3))

Attempts (5 marks)
A1 Any relevant work e.g. sets up division

(b) (ii) **5 marks** **Att 2**

$$f(x) = x^3 + 2x^2 - x - 2 = (x-1)(x^2 + 3x + 2) = (x-1)(x+1)(x+2)$$

Other factors: $(x+1)$ and $(x+2)$

* No back/ retrospective marking
* No penalty if continues to find roots - extra work
* Accept candidates quadratic from (i) if not oversimplified, see A1
* Candidates may find quadratic by comparing coefficients
* If incorrect answer at b(i) is "unfactorisable" award attempt marks at most in b(ii) unless uses relevant formula to find roots and then factors

Blunders (-3)
B1 Mathematical error e.g. finds, using formula, roots of quadratic at (i) and stops

Attempts (2 marks)
A1 If quotient at (i) is linear award attempt marks at most in (ii)

Part (c) **15 (5, 10) marks** **Att (2, 3)**

Let $g(x)=\frac{1}{x^2}-\frac{1}{2x}$ and $h(x)=1-\frac{2}{x}$, where $x\neq 0$ and $x\in\mathbb{R}$.

(i) Show that $h(x)=-2x\left[g(x)\right]$.

(ii) Find the values of x for which $g(x)=h(x)$.

(c) (i) **5 marks** **Att 2**

$$-2x[g(x)]=-2x\left(\frac{1}{x^2}-\frac{1}{2x}\right)=\frac{-2x}{x^2}+\frac{2x}{2x}=\frac{-2}{x}+\frac{1}{1}=1-\frac{2}{x}=h(x)$$

[2] [2] [5]

Blunders (-3)
B1 Mathematical error e.g. index error

Attempts (2 marks)
A1 Any relevant work e.g. correct substitution

(c) (ii) **10 marks** **Att 3**

$$\frac{1}{x^2}-\frac{1}{2x}=1-\frac{2}{x}$$

$$\Rightarrow \frac{2-x=2x^2-4x}{2x^2}$$

$$\Rightarrow 2x^2-3x-2=0$$

$$\Rightarrow (2x+1)(x-2=0$$

$$\Rightarrow x=-\tfrac{1}{2} \text{ or } x=2$$

* If equation becomes linear candidates will merit at most attempt marks

Award marks as follows:
10 marks: Fully correct answer
3 marks: Work of some merit
0 marks: Otherwise

QUESTION 3

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (15, 5) marks**	**Att (5, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a)	**15 marks**	**Att 5**

Given that $(t-1)x = 2-5t$, find the value of x when $t = 7$.

(a)	**15 marks**	**Att 5**

$(t-1)x = 2-5t \;\Rightarrow\; (7-1)x = 2-5(7)$ [9]

$\Rightarrow\; 6x = 2-35 = -33 \Rightarrow x = -5\frac{1}{2}$ or -5.5 or equivalent e.g. $\frac{-33}{6}$

[12] [15]

or

$x = \frac{2-5t}{t-1} = \frac{2-5(7)}{7-1} = -5\frac{1}{2}$ or $x = -5.5$ or equivalent e.g. $\frac{-33}{6}$

[9] [12] [15]

Blunders (-3)

B1 Mathematical error e.g. substitutes for x and solves for t

Attempts (5 marks)

A1 Any relevant work e.g. multiplies out LHS and stops or correct answer without work

Part (b) **20 (15, 5) marks** **Att (3, 2, 2)**

(i)	Solve for x and y $x - y + 5 = 0$ $x^2 + y^2 = 17.$
(ii)	Which solution gives the lesser value of $x - 2y$? Write down this value.

(b) (i) **15 (10, 5) marks** **Att (3, 2)**

Step 1 Isolation of one variable: 10 marks
Step 2 Finding second variable: 5 marks

I	II
$x - y + 5 = 0 \Rightarrow x = y - 5$	$y = x + 5$
$x^2 + y^2 = 17 \Rightarrow (y-5)^2 + y^2 = 17$	$x^2 + y^2 = 17 \Rightarrow (x+5)^2 + x^2 = 17$
$(y-5)^2 + y^2 = 17$	$(x+5)^2 + x^2 = 17$
$\Rightarrow y^2 - 10y + 25 + y^2 - 17 = 0$	$\Rightarrow x^2 + 10x + 25 + x^2 - 17 = 0$
$\Rightarrow 2y^2 - 10y + 8 = 0 \quad \Rightarrow \quad y^2 - 5y + 4 = 0$	$\Rightarrow 2x^2 + 10x + 8 = 0 \Rightarrow x^2 + 5x + 4 = 0$
$\Rightarrow (y-1)(y-4) = 0 \quad \Rightarrow \quad y = 1 \quad \text{or} \quad y = 4$	$\Rightarrow (x+4)(x+1) = 0 \quad \Rightarrow x = -4 \quad \text{or} \quad x = -1$
$y = 1 \quad \Rightarrow \quad x - 1 + 5 = 0 \quad \Rightarrow \quad x = -4$	$x = -4 \quad \Rightarrow \quad -4 - y + 5 = 0 \quad \Rightarrow \quad y = 1$
$y = 4 \quad \Rightarrow \quad x - 4 + 5 = 0 \quad \Rightarrow \quad x = -1$	$x = -1 \quad \Rightarrow \quad -1 - y + 5 = 0 \quad \Rightarrow \quad y = 4$

* Candidates may use other acceptable algebraic approaches

Blunders (-3)
B1 Mathematical error

Attempts (3 or 2 marks)
A1 Any relevant work
A2 Correct answers by trial and error or without relevant work, verified in:
- One equation – one Att 2
- Both equations – Att 3 + Att 2

If tests only one solution award Att 2 only once
A3 Some effort at a graphical solution or some effort at trial and error – one attempt only
A4 Fully correct graphical solutions merits Att 3 and Att 2 at most

(b) (ii)	**5 marks**	**Att 2**
$(-4, 1) \Rightarrow x - 2y = -4 - 2(1) = -6$	[2]	
$(-1, 4) \Rightarrow x - 2y = -1 - 2(4) = -9$	[4]	
$(-1, 4)$ gives the lesser value of -9	[5]	

* Accept candidates answer from (i) if not oversimplified. Note if only **one** answer found at (i) can earn at most att 2 for this section

Blunders (-3)
B1 Mathematical error e.g. only tests one point

Slip (-1)
S1 Incorrect or no conclusion

Attempt (2 marks)
A1 Any relevant work e.g. correct substitution for x and/or y and stops

Worthless (0)
W1 Invents values for x and y

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

(i)	Simplify $\left(\sqrt{x}-\frac{2}{\sqrt{x}}\right)\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right)$, where $x>0$ and $x \in \mathbb{R}$.
(ii)	Hence, solve $\left(\sqrt{x}-\frac{2}{\sqrt{x}}\right)\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right) = 3$, where $x>0$.
(iii)	Verify your solution.

(c) (i) **5 marks** **Att 2**

$$\left(\sqrt{x}-\frac{2}{\sqrt{x}}\right)\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right)=\left(\sqrt{x}\right)^2-\left(\frac{2}{\sqrt{x}}\right)^2 = x-\frac{4}{x} \text{ or } \frac{x^2-4}{x} \quad \text{accept either}$$

or

$$\left(\sqrt{x}-\frac{2}{\sqrt{x}}\right)\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right)=\left(\sqrt{x}\right)^2+\frac{2\sqrt{x}}{\sqrt{x}}-\frac{2\sqrt{x}}{\sqrt{x}}-\left(\frac{2}{\sqrt{x}}\right)^2 = x-\frac{4}{x} \text{ or } \frac{x^2-4}{x} \quad \text{accept either}$$

or

$$\left(\sqrt{x}-\frac{2}{\sqrt{x}}\right)\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right)=\left(\frac{x-2}{\sqrt{x}}\right)\left(\frac{x+2}{\sqrt{x}}\right)=\frac{x^2-4}{x} \text{ or } x-\frac{4}{x} \quad \text{accept either}$$

* Candidates may offer other correct methods

Blunders (-3)
B1 Mathematical error e.g. sign

Attempts (2 marks)
A1 Any relevant work e.g. starts multiplication
A2 Correct answer without work

(c) (ii) **5 marks** **Att 2**

$$\left(\sqrt{x}-\frac{2}{\sqrt{x}}\right)\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right) = 3$$

$\Rightarrow \quad x-\frac{4}{x}=3 \quad \text{or} \quad \frac{x^2-4}{x}=3$ [2]

$\Rightarrow \quad x^2-4=3x \quad \Rightarrow \quad x^2-3x-4=0$

$\Rightarrow (x-4)(x+1)=0 \quad \Rightarrow \quad x=4 \quad \text{or} \quad x=-1$ [4]

$x>0 \;\Rightarrow\; x=4$ [5]

* Accept candidates answer from (i) – if linear award attempt marks at most

Blunders (-3)
B1 Mathematical error

Slips (-1)
S1 Does not isolate correct value

Attempts (2 marks)
A1 Any relevant work

(c) (iii) **5 marks** **Att 2**

$$x = 4 \Rightarrow \left(\sqrt{4} - \frac{2}{\sqrt{4}}\right)\left(\sqrt{4} + \frac{2}{\sqrt{4}}\right) = \left(2 - \frac{2}{2}\right)\left(2 + \frac{2}{2}\right) = (1)(3) = 3$$

[2] [4] [5]

* Accept candidates answer from (ii)

* Accept verification of -1 in $\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)\left(\sqrt{x} + \frac{2}{\sqrt{x}}\right)$ - uses $\sqrt{-1} = i$

* Must verify in $\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)\left(\sqrt{x} + \frac{2}{\sqrt{x}}\right)$

Blunders (-3)
B1 Mathematical error e.g. square root error

Slips (-1)
S1 Incorrect or no conclusion, if one is required

Attempts (2 marks)
A1 Any relevant work e.g. substitutes answer from (ii) into expression and stops or verifies in simplified version.

QUESTION 4

Part (a)	**15 (5, 10) marks**	**Att (2, 3)**
Part (b)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**
Part (c)	**20 marks**	**Att (3, 3)**

Part (a) **15 (5, 10) marks** **Att (2, 3)**

Given that $6 - 4i + 3u = 5i$, where $i^2 = -1$,

(i) find *u*,

(ii) plot *u* on an Argand diagram.

(a) (i) **5 marks** **Att 2**

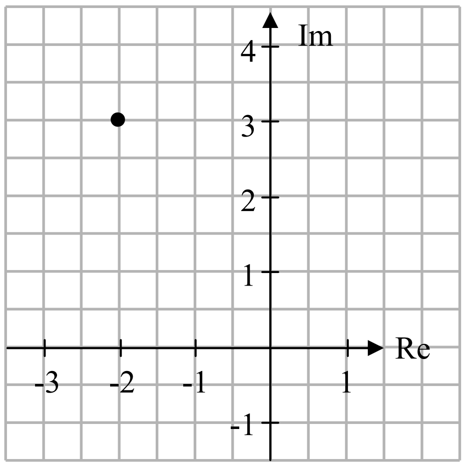
(a) (ii) **10 marks** **Att 3**

(i) $6 - 4i + 3u = 5i$

$\Rightarrow 3u = -6 + 9i \Rightarrow u = -2 + 3i$

[2] [5]

(ii)



* No penalty for interchange of real and imaginary axes if consistent

* Accept candidates answer from (i)

Blunders (-3)

B1 Mathematical error e.g. mixes up axes but check * above.

Attempts (2 or 3 marks)

A1 Any relevant work e.g. construction of axes and stops – once only

Part (b)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Let $z = 1 + i$.

(i) Find $|z|$.

(ii) Show that $z^2 + \bar{z}^2 = 0$, where $\bar{z}$ is the complex conjugate of z.

(iii) Verify that $\dfrac{1+5i}{3+2i} = z$.

(b) (i)	**5 marks**	**Att 2**

$|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$

[2] [2] [5]

* Formula not in tables

Blunders (-3)

B1 Mathematical error e.g. includes i^2 or $i^2 \neq -1$

Attempt (2 marks)

A1 Any relevant work e.g. correct formula

(b) (ii)	**5 marks**	**Att 2**

$z^2 + \bar{z}^2 = (1+i)^2 + (1-i)^2 = 1 + 2i + i^2 + 1 - 2i + i^2 = 1 + 2i - 1 + 1 - 2i - 1 = 2 - 2 = 0$

[2] [5]

* Accept "conjugate of square = square of conjugate" i.e. $\overline{z^2} = \left(\bar{z}\right)^2$

Blunders (-3)

B1 Mathematical error

Attempts (2 marks)

A1 Any relevant work e.g. correct substitution or identification of $\bar{z}$, specific or general

(b) (iii)	**5 marks**	**Att 2**

$\dfrac{1+5i}{3+2i} = \dfrac{1+5i}{3+2i} \times \dfrac{3-2i}{3-2i} = \dfrac{3-2i+15i-10i^2}{9+4} = \dfrac{13+13i}{13} = 1 + i = z.$

[2] [2] [4] [5]

or

$\dfrac{1+5i}{3+2i} = 1+i$

$\Rightarrow 1 + 5i = (3+2i)(1+i)$ [2]

$= 3(1+i) + 2i(1+i) = 3 + 3i + 2i + 2i^2 = 3 + 5i - 2 = 1 + 5i$ [5]

Blunders(-3)

B1 Mathematical error e.g. $i^2 \neq -1$

Slips (-1)

S1 Incorrect answer and no conclusion or wrong conclusion

Attempts (2 marks)

A1 Any relevant work e.g. correct substitution for z

Part (c) **20 (10, 10) marks** **Att (3, 3)**

Let $w = 3 + 4i$. Find the real numbers k and t such that $$w^2 - (k+t)w + t = 0 .$$

(c) **20 (10, 10) marks** **Att (3, 3)**

Step 1: Substitution 10 marks
Step 2: Finish 10 marks

$w^2 - (k+t)w + t = 0$

$\Rightarrow\ (3+4i)^2 - (k+t)(3-4i) + t = 0$

$\Rightarrow\ 9 + 24i - 16 - 3k - 4ki - 3t - 4ti + t = 0$

Real parts: $9 - 16 - 3k - 3t + t = 0 \quad \Rightarrow \quad -3k - 2t - 7 = 0$

Imaginary parts: $24 - 4k - 4t = 0$

$$\begin{aligned} 6k + 4t &= -14 \\ -4k - 4t &= -24 \\ 2k \quad\ &= -38 \quad \Rightarrow \quad k = -19 \end{aligned}$$

$-3k - 2t - 7 = 0 \quad \Rightarrow \quad -3(-19) - 2t - 7 = 0 \quad \Rightarrow \quad -2t = -50 \quad \Rightarrow \quad t = 25$

Step 1: Substitution

Award marks as follows:
10 marks: Fully correct substitution
7 marks: Partial correct substitution
3 marks: Attempt at substitution
0 marks: Otherwise

Step 2: Finish

Award marks as follows:
10 marks: Fully correct answer
3 marks: Work of some merit
0 marks: Otherwise

QUESTION 5

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

* Do not penalise notation

Part (a) **15 marks** **Att 5**

The n^{th} term of a sequence is $T_n = \dfrac{2n-1}{n+1}$.

Find the sum of the second and third terms of the sequence.

(a) **15 marks** **Att 5**

$$T_n = \frac{2n-1}{n+1}$$

$$T_2 = \frac{4-1}{2+1} = 1 \quad [9]$$

$$T_3 = \frac{6-1}{3+1} = \tfrac{5}{4} \quad [12]$$

$$T_2 + T_3 = 1 + \tfrac{5}{4} = 2\tfrac{1}{4} \text{ or } 2.25 \quad [15]$$

* Accept correct answers without work
* T_2 and T_3 found merits 12 marks without work

Blunders (-3)

B1 Mathematical error e.g. decimal or fractions

Attempts (5 marks)

A1 Any relevant work e.g. finds T_1 and stops

Part (b) **20 (10, 10) marks** **Att (3, 3)**

The first term of an arithmetic series is 2 and the eighth term is 30.
(i) Find T_3, the third term of the series.
(ii) Find S_{10}, the sum of the first ten terms of the series.

* Answers to parts of question must be clearly identified otherwise order in paper

(i) **10 marks** **Att 3**

$a = 2, \quad T_8 = a + 7d = 30 \quad \Rightarrow \quad 2 + 7d = 30 \quad \Rightarrow \quad 7d = 28 \quad \Rightarrow \quad d = 4$

[3] [4] [7]

$T_3 = a + 2d = 2 + 2(4) = 10$ [10]

or

$T_1 = 2, \quad T_2 = 2 + 4 = 6, \quad T_3 = 6 + 4 = 10$

[3] [7] [10]

* Accept correct answer without work
* Accept difference, *d*, as 4 without work for 7 marks

Blunders (-3)
B1 Mathematical error e.g. incorrect term found

Attempts (3 marks)
A1 Any relevant work e.g. writes $a = 2$ or $T_1 = 2$ or similar

Worthless (0)
W1 Treats as a geometric series but check for A1

(ii) **10 marks** **Att 3**

I $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$\Rightarrow S_{10} = \frac{10}{2}\left(2(2) + (10-1)4\right) = 5\left(4 + 36\right) = 200$

[4] [7] [10]

or

II 2 + 6 + 10 + 14 + 18 + 22 + 26 + 30 + 34 + 38 = 200

[7] [10]

* Accept correct answer without work
* No back/retrospective marking

Blunders (-3)
B1 Mathematical error e.g. each missing or extra term in method II

Attempts (3 marks)
A1 Writes $a = 2$ in this part

Worthless (0)
W1 Treats as a geometric series but check for A1

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

The n^{th} term of a series is $T_n = \dfrac{2}{3^{n+1}}$. **(i)** Write, in terms of n, an expression for T_{n-1}, the $(n-1)^{\text{st}}$ term. **(ii)** Prove that the series is geometric. **(iii)** Show that $S_9 = \dfrac{1}{3} - \dfrac{1}{3^{10}}$, where S_9 is the sum of the first nine terms of the series.

* Answers to parts of question must be clearly identified otherwise order in paper

(c) (i) **5 marks** **Att 2**

$$T_n = \frac{2}{3^{n+1}} \quad \Rightarrow \quad T_{n-1} = \underset{[2]}{\frac{2}{3^{n-1+1}}} = \underset{[5]}{\frac{2}{3^n}}$$

* Accept correct answer without work

Blunders (-3)
B1 Mathematical error

Attempts (3 marks)
A1 Any relevant work e.g. finds T_1 and stops

(c) (ii) **5 marks** **Att 2**

$$\underset{[2]}{\frac{T_n}{T_{n-1}}} = \frac{2}{3^{n+1}} \div \frac{2}{3^n} = \frac{2}{3^{n+1}} \times \frac{3^n}{2} = \underset{[5]}{\frac{1}{3}},$$ a constant. Thus, the series is geometric.

* Accept other correct versions of proof

Blunders (-3)
B1 Mathematical error
B2 Proves for specific terms only

Attempts (2 marks)
A1 Writes out the correct value of at least one term e.g. T_1 (at this part)

Worthless (0)
W1 Treats as an arithmetic series but see A1

(c) (iii) **5 marks** **Att 2**

$$a = T_1 = \frac{2}{3^{1+1}} = \frac{2}{9} \qquad \text{and/or} \qquad r = \frac{1}{3}$$

[2] [2]

$$S_9 = \frac{\frac{2}{9}\left(1-\frac{1}{3^9}\right)}{1-\frac{1}{3}} = \frac{1}{3}\left(1-\frac{1}{3^9}\right) = \frac{1}{3}-\frac{1}{3^{10}} \qquad [5]$$

* No back/retrospective marking

* Accept proof by correct lists method

Blunders (-3)

B1 Mathematical error e.g. indices error

Attempts (2 marks)

A1 Any relevant work

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**

Part (a) **10 marks** **Att 3**

Let $h(x) = ax + b$, where $x \in \mathbb{R}$.
Given that $h(0) = 3$ and $h(2) = -5$, find the value of a and the value of b.

(a) **10 marks** **Att 3**

$h(x) = ax + b$

$h(0) = a(0) + b = 3 \quad \Rightarrow \quad b = 3 \qquad [7]$

$h(2) = a(2) + 3 = -5 \quad \Rightarrow \quad 2a = -8 \quad \Rightarrow \quad a = -4 \quad [10]$

* Accept correct answers for a and b without work

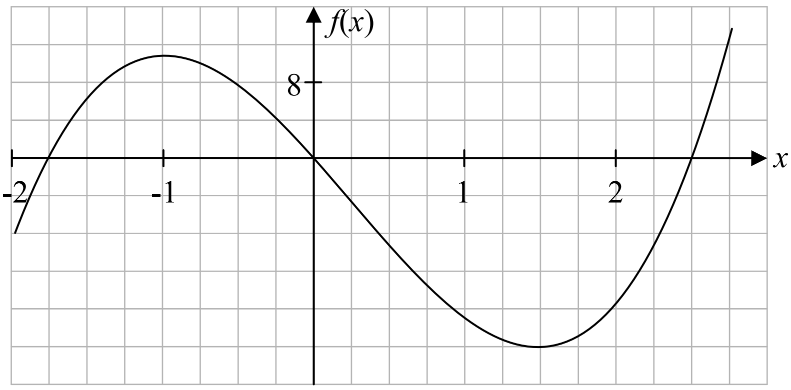
Blunders (-3)
B1 Mathematical error e.g. confuses x and $h(x)$, has $h(3) = 0$
B2 Only finds one variable i.e. a or b – accept without work

Misreading (-1)
M1 Has $h(2) = 3$ and/or $h(0) = -5$

Attempts (3 marks)
A1 Any relevant work

Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

The diagram shows part of the graph of a function *f*.



Use the graph to estimate

(i) the values of x for which $f(x) = 0$,

(ii) the values of x for which $f'(x) = 0$, where $f'(x)$ is the derivative of $f(x)$,

(iii) the range of values of x for which $f'(x) < 0$.

* Answers to parts of question must be clearly identified/labelled otherwise order in paper

* No back /retrospective marking

(b) (i)	**10 marks**	**Att 3**
(b) (ii)	**5 marks**	**Att 2**
(b) (iii)	**5 marks**	**Att 2**

(i) $x = -1.75, \quad x = 0, \quad x = 2.5$

[4] [7] [10]

(ii) $x = -1, \quad x = 1.5$

[2] [5]

(iii) $-1 < x < 1.5$

* Answer on enclosed paper or copies graph: only, award full marks if answers clearly identified otherwise attempt marks at most.

* Accept candidates answer in part (ii) for part (iii) assuming part (ii) merited at least attempt marks

Blunders (-3)

B1 Each incorrect or missing value in (i) and (ii) apply a tolerance ±0.2

B2 Mathematical error e.g. in forming inequality in (iii)

Slips (-1)

S1 Includes equal sign in inequality once only

Attempts (2 or 3 marks)

A1 Some relevant work e.g. in (ii) mentions maximum and/or minimum or in (iii) mentions *decreasing*

A2 Some valid attempt at forming *f*(*x*) to use in (ii)

Worthless (0)

W1 Copies diagram from paper and stops

Part (c) **20 (5, 10, 5) marks** **Att (2, 3, 2)**

Let $g(x) = x(3x^2 - 9)$, where $x \in \mathbb{R}$. **(i)** Find $g'(x)$, the derivative of $g(x)$. **(ii)** Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = g(x)$. **(iii)** Draw the graph of the function $g'(x)$, the derivative of $g(x)$, in the domain $-2 \le x \le 2$.

(c) (i) **5 marks** **Att 2**

I $\quad g(x) = x(3x^2 - 9) = 3x^3 - 9x \quad \Rightarrow \quad g'(x) = 9x^2 - 9$

[2] [5]

or

II $\quad g(x) = x(3x^2 - 9)$

Let $u(x) = x \quad \Rightarrow \quad u'(x) = 1$ [2]

$v(x) = 3x^2 - 9 \quad \Rightarrow \quad v'(x) = 6x$ [2]

$g'(x) = (3x^2 - 9)(1) + (x)(6x) = 3x^2 - 9 + 6x^2 = 9x^2 - 9$ [5]

Blunders (-3)

B1 Mathematical error e.g. multiplication method I

B2 Differentiate error once per part

Attempts (2 marks)

A1 Some relevant work e.g. identifies *u* and /or *v* and stops

(c) (ii) **10 marks** **Att 3**

$g'(x) = 0 \quad \Rightarrow \quad 9x^2 - 9 = 0$ [3]

$\Rightarrow \quad 9(x+1)(x-1) = 0 \quad \Rightarrow \quad x = -1$ or $x = 1$

[4] [7]

or $\Rightarrow x^2 - 1 = 0 \quad \Rightarrow (x+1)(x-1) = 0 \quad \Rightarrow \quad x = -1$ or $x = 1$

[4] [7]

$g(-1) = -1(3-9) = 6 \quad \Rightarrow \quad (-1, 6)$

$g(1) = 1(3-9) = -6 \quad \Rightarrow \quad (1, -6)$

[9] [10]

* Not required to identify local maximum and/or local minimum

* Only one point found [4] marks e.g. only uses $x = 1$

Blunders (-3)

B1 Mathematical error e.g. fails to find *g(x)* value

Attempts (3 marks)

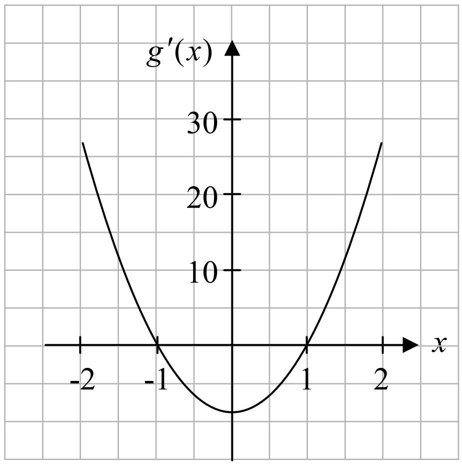
A1 Mentions local maximum/minimum occurs when $g'(x) = 0$ or mentions “turning points”

A2 Draws a graph of $g(x)$ in this part – must have a domain of $-2 \le x \le 2$

A3 Reads from a graph only (no calculus used)

(c) (iii) **5 marks** **Att 2**

$$g'(x) = 9x^2 - 9$$
$$g'(-2) = 9(-2)^2 - 9 = 27$$
$$g'(-1) = 9(-1)^2 - 9 = 0$$
$$g'(0) = 9(0) - 9 = -9$$
$$g'(1) = 9(1)^2 - 9 = 0$$
$$g'(2) = 9(2)^2 - 9 = 27$$



* No penalty if domain extended

Blunders (-3)
B1 Mathematical error e.g. incomplete domain or plots incorrect function correctly e.g. $g(x)$

Slips (-1)
S1 Each missing or incorrectly plotted point to a maximum of 3 slips

Attempts (2 marks)
A1 Any relevant work e.g. sets up grid on graph paper

QUESTION 7

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **15 marks** **Att 5**

Differentiate $y = 6x - x^2 - 5x^4$ with respect to x.

(a) **15 marks** **Att 5**

$$y = 6x - x^2 - 5x^4 \Rightarrow \frac{dy}{dx} = 6 - 2x - 20x^3$$

* Correct answer without work or notation: full marks.
* If done from first principles, ignore errors in procedure – just mark the answer.
* Only one non zero term correct, award 9 marks.

Blunders (-3)

B1 Mathematical error e.g. differentiation error once per term – includes sign

Attempts (5 marks)

A1 Any relevant step e.g. mentions $\frac{dy}{dx}$ or $f'(x)$

A2 A correct coefficient or a correct index of x in one of the term(s)/part(s)

Part (b)	**20 (10, 10) marks**	**Att (3, 3)**

(i) Differentiate $y = (3x^2+2)(x^3-x)$ with respect to x.

(ii) Given that $y = (x^3-2x^2+4)^5$, find the value of $\frac{dy}{dx}$ when $x=-1$.

(b) (i)	**10 marks**	**Att 3**

I $\quad y = (3x^2+2)(x^3-x)$

Let $u = 3x^2+2$ [3] $\Rightarrow \frac{du}{dx} = 6x$ [4]

Let $v = x^3 - x$ [3] $\Rightarrow \frac{dv}{dx} = 3x^2 - 1.$ [4]

$\frac{dy}{dx} = (3x^2+2)(3x^2-1) + (x^3-x)(6x)$ [10]

$= 6x^4 - 6x^2 + 9x^4 + 6x^2 - 3x^2 - 2 \quad = 15x^4 - 3x^2 - 2$

or

I $\quad y = (3x^2+2)(x^3-x) = 3x^5 - 3x^3 + 2x^3 - 2x$ [4]

$\Rightarrow y = 3x^5 - x^3 - 2x$

$\frac{dy}{dx} = 15x^4 - 3x^2 - 2$ or $\frac{dy}{dx} = 15x^4 - 3x^2 - 2 = 15x^4 - 9x^2 + 6x^2 - 2$ [10]

* Check list of blunders re use of formulae

Blunders (-3)

B1 Differentiation errors, once per part

B2 Multiplication, method II, once only unless oversimplified. (Must contain two terms)

Attempts (3 marks)

A1 Any relevant work e.g. u and/or v correctly identified and stops

(b) (ii) **10 marks** **Att 3**

I $\quad y = \left(x^3 - 2x^2 + 4\right)^5$

$\frac{dy}{dx} = 5\left(x^3 - 2x^2 + 4\right)^4\left(3x^2 - 4x\right)$ [9]

$x = -1: \quad \frac{dy}{dx} = 5\left((-1)^3 - 2(-1)^2 + 4\right)^4\left(3(-1)^2 - 4(-1)\right) = 5\left(-1 - 2 + 4\right)^4\left(7\right) = 35$ [10]

or

$y = \left(x^3 - 2x^2 + 4\right)^5$

II $\quad$ Let $u = x^3 - 2x^2 + 4 \Rightarrow \frac{du}{dx} = 3x^2 - 4x$

$y = u^5 \Rightarrow \frac{dy}{du} = 5u^4$ [4]

$\frac{dy}{dx} = 5u^4\left(3x^2 - 4x\right) = 5\left(x^3 - 2x^2 + 4\right)^4\left(3x^2 - 4x\right)$ [9]

$x = -1: \quad \frac{dy}{dx} = 5\left((-1)^3 - 2(-1)^2 + 4\right)^4\left(3(-1)^2 - 4(-1)\right) = 5\left(-1 - 2 + 4\right)^4\left(7\right) = 35$ [10]

* Accept $\frac{dy}{dx} = 5\left((-1)^3 - 2(-1)^2 + 4\right)^4\left(3(-1)^2 - 4(-1)\right) = 5\left(-1 - 2 + 4\right)^4\left(7\right) = 35$ for full marks

Blunder (-3)
B1 Differentiation errors once per part

Attempts (3 marks)
A1 Any relevant work e.g. relevant differentiation

Worthless (0)
W1 Substitutes $x = -1$ into y and stops or evaluates y

Part (c)	15 (5, 5, 5) marks	Att (2, 2, 2)

A ball is thrown vertically down from the top of a high building.
The distance, s metres, the ball falls is given by

$$s = 3t + 5t^2$$

where t is the time in seconds from the instant the ball is thrown.

(i) Find the speed of the ball after 3 seconds.

(ii) Find the time t when the ball is falling at a speed of 23 ms^{-1}.

(iii) The ball hits the ground at a speed of 38 ms^{-1}.
How high is the building?

* Units: Penalise as per guidelines
* No retrospective marking
* If parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c)(i) to (c)(iii)

(c) (i)	5 marks	Att 2

$$\frac{ds}{dt} = 3 + 10t = 3 + 30 = 33 \text{ m s}^{-1} \text{ at } t = 3$$

[2] [5]

Blunders (-3)
B1 Mathematical error e.g. differentiation error

Slip(-1)
S1 No units or incorrect units only applies if answer numerically correct e.g. 33 (no units)

Attempts (2 marks)
A1 Any correct relevant work e.g. $\frac{ds}{dt}$ or $\frac{dy}{dx}$ mentioned

Worthless (0)
W1 No differentiation e.g. t = 3 substituted into original equation

(c) (ii) **5 marks** **Att 2**

$$\frac{ds}{dt} = 3 + 10t = 23 \Rightarrow 10t = 20 \Rightarrow t = 2 \text{ seconds}$$

[2] [5]

* Must use $\frac{ds}{dt}$ explicitly at this part

* Accept candidates $\frac{ds}{dt}$ from (i) if involved differentiation

Blunders(-3)

B1 Mathematical error e.g. solving equation

Slips (-1)

S1 No units or incorrect units

Attempts (2 marks)

A1 Any relevant work

Worthless (0)

W1 No differentiation e.g. solves $s = 23$

W2 $\frac{d^2s}{dt^2} = 10$ only

(c) (iii) **(5) marks** **Att 2**

$$\frac{ds}{dt} = 3 + 10t = 38 \Rightarrow 10t = 35 \Rightarrow t = 3.5$$

$$s = 3t + 5t^2 = 3(3.5) + 5(3.5)^2 = 10.5 + 61.25 = 71.75 \text{ m}$$

* Must use $\frac{ds}{dt}$ explicitly at this part

* Accept candidates $\frac{ds}{dt}$ from (i) if involved differentiation

Blunders (-3)

B1 Mathematical error

Slips (-1)

S1 No units or incorrect units

Attempts (2 marks)

A1 Any relevant work e.g. uses s as height.

QUESTION 8

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **15 marks** **Att 5**

Let $g(x) = k(1-x)$, where $x \in \mathbb{R}$.
Given that $g(-5) = 20$, find the value of k.

(a) **15 marks** **Att 5**

$g(x) = k(1-x)$

$g(-5) = k(1+5) = 20 \quad \Rightarrow \quad 6k = 20 \quad \Rightarrow \quad k = \frac{20}{6} = \frac{10}{3}$ or $3\frac{1}{3}$ or equivalent

[9] [12] [15]

* Accept correct answer without work.

Blunder (-3)

B1 Mathematical error e.g. solves $g(20) = -5$ or substitutes for k then solves correctly for x

Attempts (5 marks)

A1 Some relevant work e.g. multiplies out RHS and stops

Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Let $f(x)=\dfrac{5+x^2}{2-x}$, where $x\in\mathbb{R}$ and $x\neq 2$.

(i) Find $f(5)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Show that $f'(x)=0$ at $x=-1$.

(b) (i)	**10 marks**	**Att 3**

$$f(x)=\frac{5+x^2}{2-x}\quad\Rightarrow\quad f(5)=\underset{[7]}{\frac{5+5^2}{2-5}}=\frac{5+25}{2-5}=\underset{[10]}{\frac{30}{-3}}=-10$$

Blunders (-3)

B1 Mathematical error e.g. solves $f(x)=5$

Attempts (3 marks)

A1 Some relevant work e.g. work at substitution

(b) (ii)	**5 marks**	**Att 2**

I or II

I

$f(x)=\dfrac{5+x^2}{2-x}$

Let $u(x)=5+x^2 \Rightarrow u'(x)=2x$

Let $v(x)=2-x \Rightarrow v'(x)=-1$

$f'(x)=\dfrac{(2-x)(2x)-(5+x^2)(-1)}{(2-x)^2}$ [5]

$=\dfrac{4x-2x^2+5+x^2}{(2-x)^2}=\dfrac{-x^2+4x+5}{(2-x)^2}$

II

$f(x)=(5+x^2)(2-x)^{-1}$

Let $u(x)=5+x^2 \Rightarrow u'(x)=2x$

Let $v(x)=(2-x)^{-1} \Rightarrow v'(x)=-1(2-x)^{-2}(-1)$

$f'(x)=2x(2-x)^{-1}+(5+x^2)(2-x)^{-2}$ [5]

* See blunder list at start on use of calculus formulae

Blunders (-3)

B1 Mathematical error

Attempts (2 marks)

A1 Some relevant work e.g. identifies u and/or v and stops

(b) (iii) **5 marks** **Att 2**

$$f'(x) = \frac{-x^2 + 4x + 5}{(2-x)^2}$$

$$x = -1: \quad f'(-1) = \underbrace{\frac{-(-1)^2 + 4(-1) + 5}{(2-(-1))^2}}_{[2]} = \frac{-1-4+5}{(2+1)^2} = \underbrace{\frac{0}{9}}_{[4]} = \underbrace{0}_{[5]}$$

* Accept candidates $f'(x)$ from (ii), penalise error in simplification at this stage

Blunders (-3)
B1 Mathematical errors e.g. simplification of $f'(x)$ in (ii) or incorrect substitution in $f'(x)$

Slip (-1)
S1 Incorrect or no conclusion if not proven

Attempts (2 marks)
A1 Some relevant work e.g. uses $f(x)$ instead of $f'(x)$ and substitutes -1

Worthless (0)
W1 No effort at substitution

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

Let $h(x) = 5 + 3x - x^2$, where $x \in \mathbb{R}$.

(i) Find the co-ordinates of the point P at which the curve $y = h(x)$ cuts the y-axis.

(ii) Find the equation of the tangent to the curve $y = h(x)$ at P.

(iii) The tangent to the curve $y = h(x)$ at $x = t$ is perpendicular to the tangent at P. Find the value of t.

(c) (i) **5 marks** **Att 2**

Cuts y-axis when $x = 0$ [2]

$h(x) = 5 + 3x - x^2 \Rightarrow h(0) = 5 + 3(0) - (0)^2 = 5 \quad \Rightarrow P(0, 5)$

[2] [4] [5]

* Accept (0, 5) without work

Blunders (-3)

B1 Mathematical error e.g. uses incorrect axis

Attempts (2 marks)

A1 Some relevant work e.g. states $y = 5$ without work

(c) (ii) **5 marks** **Att 2**

$h(x) = 5 + 3x - x^2 \quad \Rightarrow \quad h'(x) = 3 - 2x \quad \Rightarrow \quad h'(0) = 3 - 2(0) = 3$

Equation of tangent: $y - 5 = 3(x - 0)$ or $3x - y + 5 = 0$

* Accept candidates value for P from (i) if not oversimplified- No retrospective marking

* No differentiation merits attempt marks at most

Blunders (-3)

B1 Mathematical error e.g. solves $h'(x) = 0$ and continues i.e. incorrect slope

Attempts (2 marks)

A1 Some relevant work e.g. connects slope of tangent with differentiation or equation of line formula with some correct substitution

(c) (iii) **5 marks** **Att 2**

$h(x) = 5 + 3x - x^2 \quad \Rightarrow \quad h'(x) = 3 - 2x$

Slope of tangent at $P = 3 \quad \Rightarrow \quad$ slope of perpendicular tangent = $-\frac{1}{3}$ [2]

$h'(t) = 3 - 2t = -\frac{1}{3} \quad \Rightarrow \quad -2t = -\frac{10}{3} \quad \Rightarrow \quad t = \frac{5}{3}$ or equivalent [5]

* Accept candidate's value of slope from (ii) if not oversimplified

Blunders (-3)

B1 Mathematical error

Attempts (2 marks)

A1 Some relevant work e.g. mentions $m_1 \times m_2 = -1$

2012. M128

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2012

Mathematics
(Project Maths – Phase 1)

Paper 2

Ordinary Level

Monday 11 June   Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

Note: the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	125 marks	5 questions
Section B	Contexts and Applications	125 marks	2 questions
Section C	Area and Volume (old syllabus)	50 marks	1 question

Answer **all eight** questions, as follows:

In Section A, answer:

Questions 1 to 4 and

either Question 5A **or** Question 5B.

In Section B, answer Questions 6 and 7.

In Section C, answer Question 8.

Write your answers in the spaces provided in this booklet. You will lose marks if you do not do so. There is space for extra work on the back cover of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	125 marks

Answer **all five** questions from this section.

Question 1 **(25 marks)**

Peter and Niamh go to a large school. One morning, they arrive early. While they are waiting, they decide to guess whether each of the next three students to come in the door will be a boy or a girl.

(a) Write out the sample space showing all the possible outcomes. For example, BGG is one outcome, representing Boy, Girl, Girl.

> BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG

(b) Peter says these outcomes are equally likely. Niamh says they are not. What do you need to know about the students in the school to decide which of them is correct?

> The number of boys and the number of girls in the school.

(c) If all the outcomes are equally likely, what is the probability that the three students will be two girls followed by a boy?

> P(GGB) = $\frac{1}{8}$ or 0·125 or 12·5%

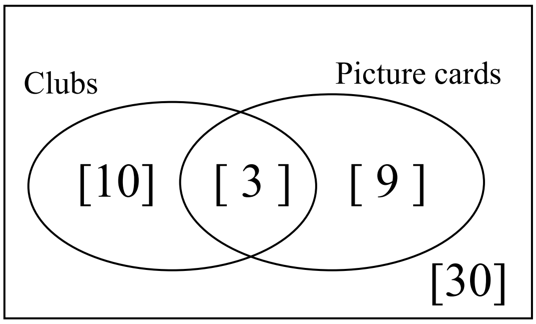
(d) Niamh guesses that there will be at least one girl among the next three students. Peter guesses that the next three students will be either three boys or two boys and a girl.
Who is more likely to be correct, assuming all outcomes are equally likely?
Justify your answer.

> P(at least one girl) $= \frac{7}{8}$ or 0·875 or 87·5%
>
> P(three boys or two boys and a girl $= \frac{4}{8}$ or $\frac{1}{2}$ or 0·5 or 50%
>
> Niamh is more likely to be correct because of the greater probability.

Question 2 **(25 marks)**

(a) In the Venn diagram below, the universal set is a normal deck of 52 playing cards. The two sets shown represent *clubs* and *picture cards* (kings, queens and jacks).

Show on the diagram the number of elements in each region.



(b) **(i)** A card is drawn from a pack of 52 cards.
Find the probability that the card drawn is the king of clubs.

P(king of clubs) $= \frac{1}{52}$

(ii) A card is drawn from a pack of 52 cards.
Find the probability that the card drawn is a club or a picture card.

P(club or picture card) $= \frac{22}{52} = \frac{11}{26}$

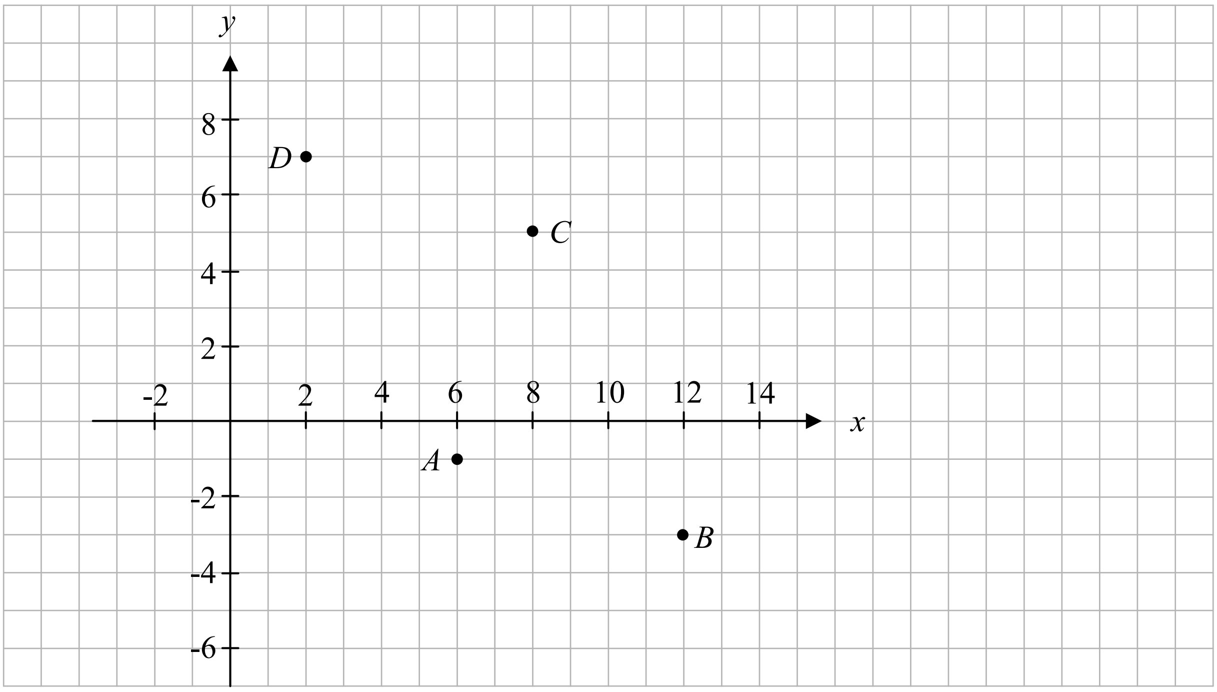
(iii) Two cards are drawn from a pack of 52 cards. Find the probability that neither of them is a club or a picture card. Give your answer correct to two decimal places.

P(not club or picture card) $= \frac{30}{52} \times \frac{29}{51} \approx 0{\cdot}33$

Question 3 **(25 marks)**

$A(6,-1)$, $B(12,-3)$, $C(8,5)$ and $D(2,7)$ are four points.

(a) Plot the four points on the diagram below.



(b) Describe two different ways of showing, using co-ordinate geometry techniques, that the points form a parallelogram *ABCD*.

Any TWO of:

- Show that opposite sides are parallel by showing the slopes of opposite lines are equal.
- Show that the diagonals bisect each other by showing the midpoint of [*AC*] equals the midpoint of [*DB*].
- Show that the opposite sides are equal in length using the length formula.
- Show that $\overrightarrow{AB}$ maps D onto C or similar.

(c) Use one of the ways you have described to show that *ABCD* is a parallelogram.

Slope $AB = \frac{-3+1}{12-6} = -\frac{2}{6}$, Slope $DC = \frac{7-5}{2-8} = -\frac{2}{6}$ $\Rightarrow AB \parallel DC$

Slope $BC = \frac{5+3}{8-12} = -2$, Slope $AD = \frac{7+1}{2-6} = -2$ $\Rightarrow BC \parallel AD$

Hence, *ABCD* a parallelogram

or

Midpoint $[AC] = (\frac{6+8}{2}, \frac{-1+5}{2}) = (7,2)$, Midpoint $[BD] = (\frac{12+2}{2}, \frac{-3+7}{2}) = (7,2)$,

$\Rightarrow$ Diagonals bisect. Hence, *ABCD* a parallelogram

or

Length $[AB] = \sqrt{(12-6)^2 + (-3+1)^2} = \sqrt{40}$

Length $[DC] = \sqrt{(2-8)^2 + (7-5)^2} = \sqrt{40}$

Length $[AD] = \sqrt{(6-2)^2 + (-1-7)^2} = \sqrt{80}$

Length $[BC] = \sqrt{(12-8)^2 + (-3-5)^2} = \sqrt{80}$

$\Rightarrow$ Opposite sides are equal. Hence, *ABCD* a parallelogram

or

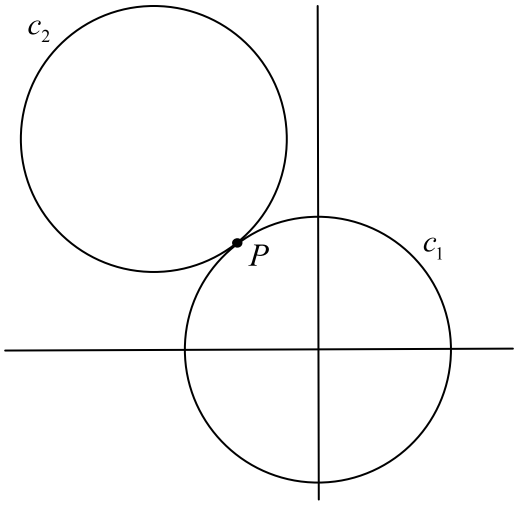
$A(6,-1) \rightarrow B(12,-3)$ maps $D(2,7) \rightarrow (2+6, 7-2) = C(8,5)$ or similar

$\Rightarrow \overrightarrow{AB} = \overrightarrow{DC}$. Hence, *ABCD* a parallelogram

Question 4 **(25 marks)**

The diagram shows two circles c_1 and c_2 of equal radius.

c_1 has centre (0, 0) and it cuts the *x*-axis at (5, 0).



(a) Find the equation of c_1.

$x^2 + y^2 = 5^2 = 25$

(b) Show that the point $P(-3, 4)$ is on c_1.

$x^2 + y^2 = (-3)^2 + 4^2 = 9 + 16 = 25 = r^2$

(c) The two circles touch at $P(-3, 4)$.
P is on the line joining the two centres.
Find the equation of c_2.

$(0, 0) \to (-3, 4)$ maps $(-3, 4) \to (-6, 8)$

$c_2: \quad (x+6)^2 + (y-8)^2 = 25$

(d) Find the equation of the common tangent at P.

Slope of line of centres: $\dfrac{8-0}{-6-0} = -\dfrac{4}{3}$

Perpendicular slope, the slope of the tangent: $\dfrac{3}{4}$

Equation of tangent: $y - 4 = \frac{3}{4}(x+3) \Rightarrow 3x - 4y + 25 = 0$

Question 5 **(25 marks)**

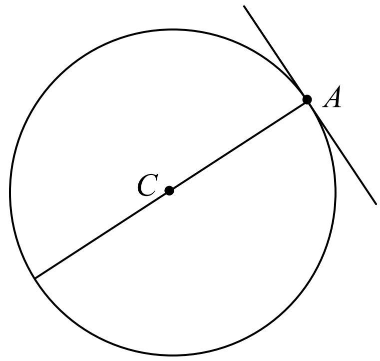
Answer **either** 5A **or** 5B.

Question 5A

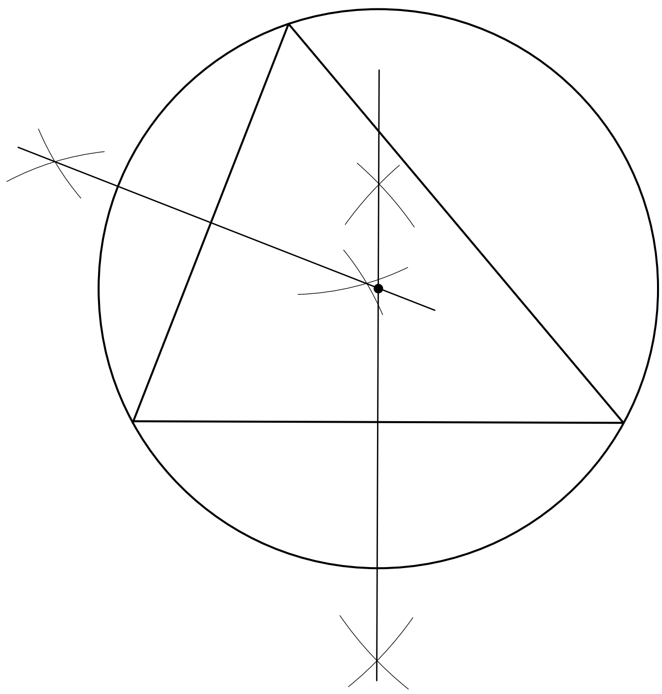
(a) **(i)** Write down a geometrical result that can be used to construct a tangent to a circle at a point.

The tangent is perpendicular to the radius at the point of contact

(ii) On the diagram shown, construct the tangent to the circle at *A*.



(b) Construct the circumcentre and circumcircle of the triangle below, using only a straight edge and compass. Show all construction marks clearly.



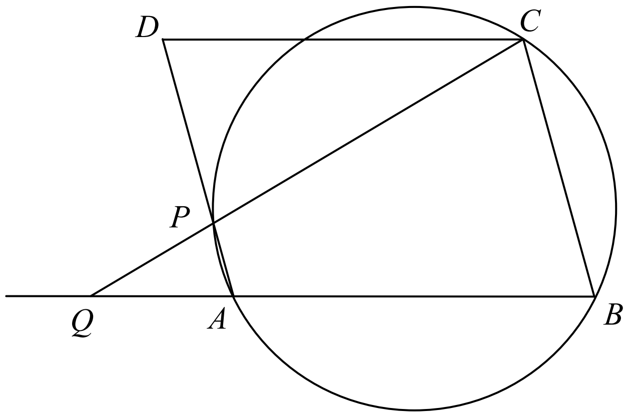
OR

Question 5B

ABCD is a parallelogram.

The points *A*, *B* and *C* lie on the circle which cuts [*AD*] at *P*.
The line *CP* meets the line *BA* at *Q*.

Prove that $|CD| = |CP|$.



$|\angle ABC| = |\angle CDP|$, opposite angles in a parallelogram

$|\angle ABC| + |\angle CPA| = 180°$, opposite angles in cyclic quadrilateral

$|\angle DPC| + |\angle CPA| = 180°$

Hence, $|\angle ABC| = |\angle DPC|$

Hence, $|\angle CDP| = |\angle DPC| \Rightarrow$ isosceles triangle or $|CD| = |CP|$

Section B	Contexts and Applications	125 marks

Answer Question 6 and Question 7.

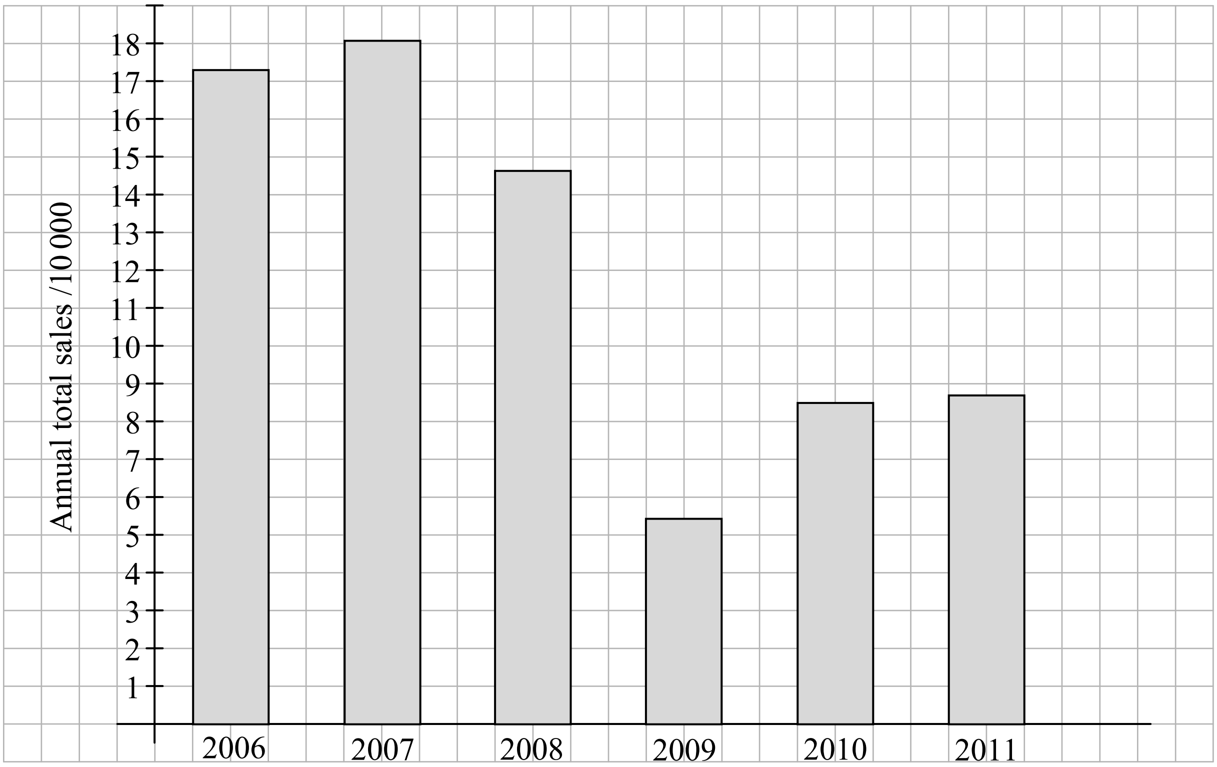
Question 6 **(75 marks)**

The following table gives data on new private cars sold in Ireland in each quarter of each year from 2006 to 2011.

New private cars sales								
Number of cars sold						Engine type of cars sold		
Year	January to March	April to June	July to Sept.	October to Dec.	Annual Total	Petrol	Diesel	Other
2006	75 769	54 572	32 873	10 059	173 273	128 634	44 010	629
2007	81 750	57 124	32 418	9 462	180 754	128 346	50 560	1 848
2008	77 441	37 128	27 361	4 540	146 470	92 298	50 283	3 889
2009	27 140	15 225	9 049	3 018	54 432	22 802	30 645	985
2010	34 555	26 806	17 011	6 535	84 907	27 124	53 998	3 785
2011	39 484	29 770	13 467	4 211	86 932	23 246	61 730	1 956

(Source: Central Statistics Office, http://www.cso.ie)

(a) **(i)** Show the *annual total* sales of cars over the six years, using a suitable chart.



(ii) Find the mean number of cars sold per year over the six years.

Mean $\frac{1}{6}(173273+180754+146470+54432+84907+86932)=\frac{1}{6}(726768)=121128$

(iii) Calculate the percentage increase in annual car sales between 2009 and 2011.

$$\text{Increase} = 86\,932 - 54\,432 = 32\,500 \Rightarrow \%\text{ increase} = \frac{32500}{54432} \times 100 = 59{\cdot}71\%$$

(iv) Aoife says that this increase shows car sales are currently going well. Paul says that car sales are currently going badly. He says that sales have fallen by 52% since 2007 and that they are well below average.
Complete the sentences below to give a criticism of each argument.

Aoife's argument does not recognise that this increase is from a very low base, and that sales had been much better before 2009.

Paul's argument does not recognise that, although sales are much lower now than in 2007, they have recovered a lot since their lowest point in 2009.

(v) Give a more balanced description of the pattern of car sales over the six years.

Sales fell dramatically from 2007 to 2009; they recovered a lot since then, but are still much lower than they were at the start.

(b) **(i)** Describe how the sales of the cars are distributed over the four quarters of each year.

Highest quarterly sales are in the first quarter and decrease significantly in each subsequent quarter.

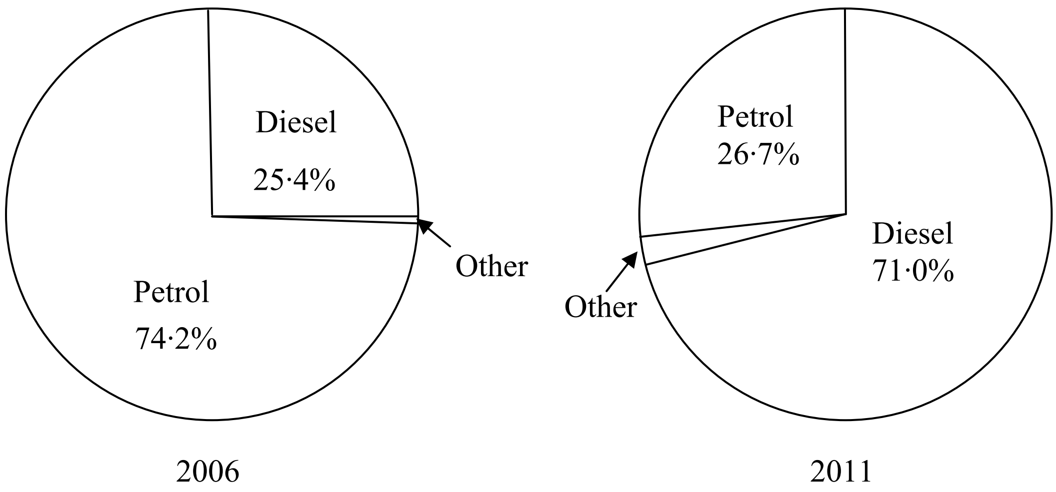
(ii) Suggest a reason for this pattern of sales.

People like to buy new cars early in the year so that they have a new year number plate.

(iii) The sales for the first quarter of 2012 are 36 081.
Find, with justification, an estimate for the total annual sales for 2012.

First quarter sales in 2012 are about one-third of the range between those quarters in 2010 and 2011. Assuming annual sales will retain that proportion would give an estimate of 85 500 for annual sales.

(c) (i) Two pie charts are being used to show the change from 2006 to 2011 in the popularity of petrol and diesel cars. Complete the second pie chart.



$$\text{Diesel} = \frac{61730}{86932} \times 360 = 256^\circ$$

$$\text{Petrol} = \frac{23246}{86932} \times 360 = 96^\circ$$

$$\text{Other} = \frac{1956}{86932} \times 360 = 8^\circ$$

(ii) Which of the following statements best describes the change over time in the popularity of diesel cars as a percentage of the total?

A. Diesel cars have suddenly become very popular in the last year or two.

B. Diesel cars have increased very steadily in popularity over the last six years.

C. Diesel cars have become very popular since car sales started to improve.

D. Diesel cars got more popular each year, with an especially big increase in 2009.

E. Diesel cars became popular as car sales fell but have been getting less popular as they rise again.

Write the letter corresponding to the correct answer in the box. D

(d) A survey of some of the most popular models of private cars sold in 2011 examined the CO_2 emissions in g/km from diesel engines and petrol engines. The data are as follows:

Diesel engines	Petrol engines
117, 125, 120, 125, 134, 110, 118, 114, 119, 119, 116, 107.	139, 133, 150, 157, 138, 159, 129, 138, 134, 129, 129, 136.

(i) Construct a back-to-back stem-and-leaf plot of the above data.

Diesel	Stem	Petrol
7	10	
9, 9, 8, 7, 6, 4, 0	11	
5, 5, 0	12	9, 9, 9
4	13	3, 4, 6, 8, 8, 9
	14	
	15	0, 7, 9

Key: 12| 9 means 129

(ii) Does the information suggest that diesel engines produce lower CO_2 emissions than petrol engines? In your answer you should refer to the stem-and-leaf plot and to an appropriate measure of central tendency.

Yes. The diesel engines grouped at the top of the plot have a smaller median [/mean] value.

(iii) Does the information suggest that there is a greater variation in the CO_2 emissions of diesel engines than petrol engines? In your answer you should refer to the stem-and-leaf plot and an appropriate measure of variability.

No. The emissions for the petrol engines are *more* spread out than for the diesel ones. The range [/interquartile range /standard deviation] for the petrol engines is greater than that for the diesel engines.

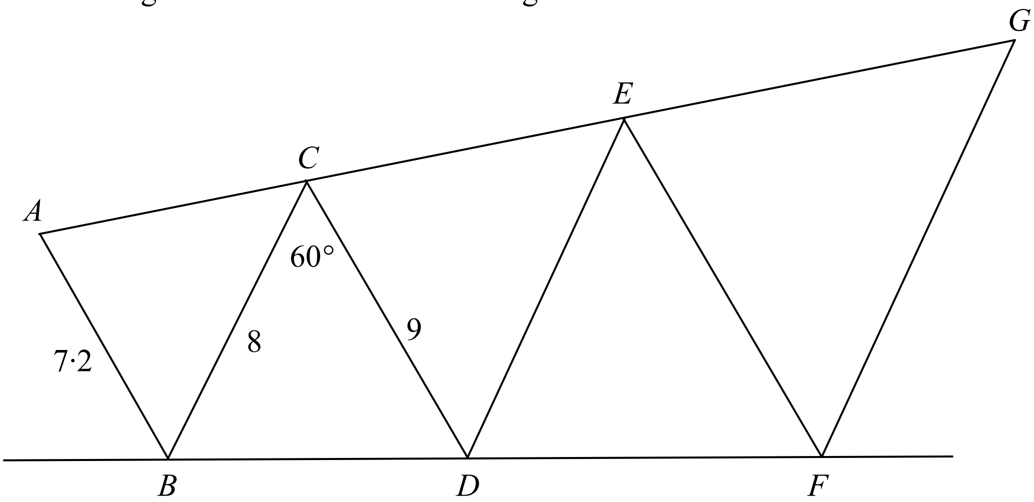
Question 7 **(50 marks)**

The planned supports for the roof of a building form scalene triangles of different sizes.

(a) Explain what is meant by a **scalene triangle**.

> A triangle in which the three sides have different lengths

The triangle *EFG* is the image of the triangle *CDE* under an enlargement and the triangle *CDE* is the image of the triangle *ABC* under the same enlargement.



The proposed dimensions for the structure are $|AB| = 7{\cdot}2$ m, $|BC| = 8$ m, $|CD| = 9$ m and $|\angle DCB| = 60°$.

(b) Find the length of [*FG*].

> Scale factor $= \frac{9}{7.2} = 1{\cdot}25$
> $|DE| = 1{\cdot}25\,|BC| = 1{\cdot}25(8) = 10 \;\Rightarrow\; |FG| = 1{\cdot}25|DE| = 1{\cdot}25(10) = 12{\cdot}5$ m

(c) Find the length of [*BD*], correct to three decimal places.

> $|BD|^2 = 8^2 + 9^2 - (2)(8)(9)\cos 60° = 64 + 81 - 72 = 73$
> $\Rightarrow \; |BD| = \sqrt{73} = 8{\cdot}544$ m

(d) The centre of the enlargement is O. Find the distance from O to the point B.

$$\frac{|OD|}{|OB|} = \frac{x + 8{\cdot}544}{x} = 1{\cdot}25$$

$$\Rightarrow \quad x + 8{\cdot}544 = 1{\cdot}25x$$

$$\Rightarrow \quad 0{\cdot}25x = 8{\cdot}544$$

$$\Rightarrow \quad x = 34{\cdot}176 \text{ m}$$

(e) A condition of the planning is that the height of the point *G* above the horizontal line *BF* cannot exceed 11·6 m.

Does the plan meet this condition? Justify your answer by calculation.

$|\angle GFH| = \alpha = |\angle CBD|$

In triangle *CBD*:

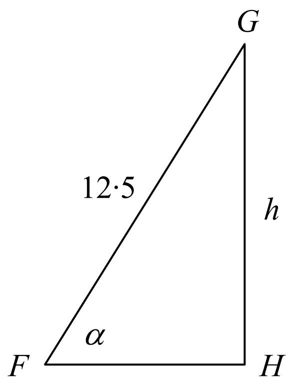
$$\frac{\sin\alpha}{9} = \frac{\sin 60^\circ}{8{\cdot}544} \Rightarrow \sin\alpha = \frac{9\sin 60^\circ}{8{\cdot}544}$$

In triangle *GFH*:

$$\sin\alpha = \frac{h}{12{\cdot}5} = \frac{9\sin 60^\circ}{8{\cdot}544}$$

$$\Rightarrow h = \frac{12{\cdot}5 \times 9\sin 60^\circ}{8{\cdot}544} = 11{\cdot}4 < 11{\cdot}6$$

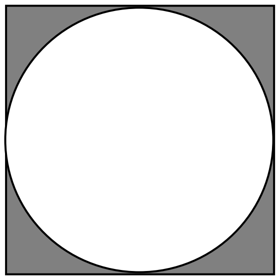
Yes, the plan meets the condition.



Section C Area and Volume (old syllabus) 50 marks

Answer Question 8 from this section.

Question 8 **(50 marks)**



(a) The diagram shows a circle inscribed in a square.
The area of the square is $16\ \text{cm}^2$.

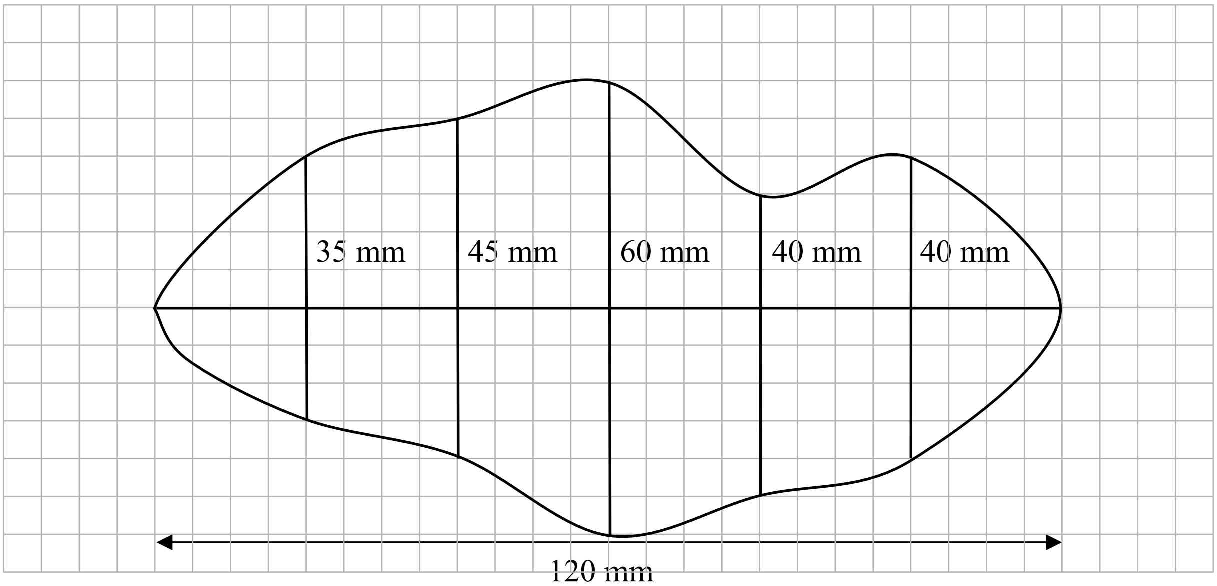
(i) Find the radius length of the circle.

$l^2 = 16 \quad \Rightarrow \quad l = 4 \quad \Rightarrow \quad \text{radius} = 2\ \text{cm}$

(ii) Find the area of the shaded region, in cm^2, correct to one decimal place.

Shaded area: $16 - \pi(2)^2 = 16 - 12{\cdot}566 = 3{\cdot}433 = 3{\cdot}4\ \text{cm}^2$

(b) In order to estimate the area of the irregular shape shown below, a horizontal line was drawn across the widest part of the shape and five offsets (perpendicular lines) were drawn at equal intervals along this line.



(i) Find the lengths of the horizontal line and the offsets, taking each grid unit as 5 mm, and record the lengths on the diagram.

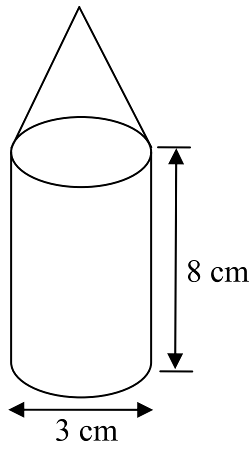
(ii) Use Simpson's rule to estimate the area of the shape.

$$
\begin{aligned}
A &= \tfrac{20}{3}\big(0 + 0 + 2(45 + 40) + 4(35 + 60 + 40)\big) \\
&= \tfrac{20}{3}\big(0 + 170 + 540\big) \\
&= \tfrac{20}{3}\big(710\big) \\
&= 4733\tfrac{1}{3}\ \text{mm}^2
\end{aligned}
$$

(c) A solid wax candle is in the shape of a cylinder with a cone on top, as shown in the diagram.

The diameter of the base of the cylinder is 3 cm and the height of the cylinder is 8 cm.

The volume of the wax in the candle is $21\pi \text{ cm}^3$.



(i) Find the height of the candle.

Volume of cylinder = $\pi(1\cdot5)^2 8 = 18\pi$
Volume of cone $= 21\pi - 18\pi = 3\pi$
$\frac{1}{3}\pi r^2 h = 3\pi \Rightarrow \frac{1}{3}\pi(1{\cdot}5)^2 h = 3\pi \Rightarrow h = \dfrac{9}{2{\cdot}25} = 4$ cm

Height of candle: 8 + 4 = 12 cm

(ii) Nine of these candles fit into a rectangular box. The base of the box is a square. Find the volume of the smallest rectangular box that the candles will fit into.

Square base $\Rightarrow$ 3 candles wide $\times$ 3 candles deep.
Dimensions of base = 3(3) $\times$ 3(3).
Area of base of box: $9 \times 9 = 81 \text{ cm}^2$

Height of box: 12 cm
Volume: $81 \times 12 = 972 \text{ cm}^3$

Marking Scheme – Paper 2, Section A, Section B and Section C

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scale	0, 5	0, 2, 5	0, 2, 4, 5	
10 mark scale		0, 5, 10	0, 4, 7, 10	0, 2, 5, 8, 10
15 mark scale			0, 5, 10, 15	0, 4, 7, 11, 15
20 mark scale				
25 mark scale			0, 8, 17, 25	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)	10C
(b)	5B
(c)	5B
(d)	5C

Question 2

(a)	5C
(b) (i)	10B
(b) (ii)	5C
(b) (iii)	5C*

Question 3

(a)	15C
(b)	5B
(c)	5C

Question 4

(a)	10C
(b)	5C
(c)	5C
(d)	5C

Question 5A

(a) (i)	5B
(a) (ii)	5B
(b)	15D

Question 5B

	25C

Section B

Question 6

(a)	(i)	10C
(a)	(ii)	10C
(a)	(iii)	5C
(a)	(iv)	5B
(a)	(v)	5B
(b)	(i)	5B
(b)	(ii)	5A
(b)	(iii)	5B
(c)	(i)	5C
(c)	(ii)	5B
(d)	(i)	5C
(d)	(ii)	5C
(d)	(iii)	5C

Question 7

(a)	5B
(b)	15D*
(c)	15D*
(d)	5B*
(e)	10D

Section C

Question 8

(a)	(i)	5B*
(a)	(ii)	5B*
(b)	(i)	10C*
(b)	(ii)	15D*
(c)	(i)	10D*
(c)	(ii)	5B*

Detailed marking notes

Section A

Question 1

(a) Scale 10C (0, 4, 7,10)

Low partial credit:

- Any work of merit e.g. one correct outcome other than BGG.

High partial credit:

- An almost correct response such as one/two outcomes missing or extra outcomes.

(b) Scale 5B (0, 2, 5)

Partial credit:

- Any work of merit e.g. the number of boys or the number of girls.

(c) Scale 5B (0, 2, 5)

Partial credit:

- Any work of merit e.g. correct numerator or denominator in fraction format e.g. P(2 girls and a boy) = $\frac{3}{8}$

Full credit:

- Correct answer without work shown.

(d) Scale 5C (0, 2, 4, 5)

Low partial credit:

- Any work of merit e.g. outcomes for Niamh or Peter given or a correct answer with no work shown.

High partial credit:

- An almost correct response such as the outcomes for both Niamh and Peter given.
- Answer with one element missing.

Question 2

(a) Scale 5C (0, 2, 4, 5)

Low partial credit:

- Any work of merit such as one correct element.

High partial credit:

- An almost correct response such as two or three correct elements.

(b)(i) Scale 10B (0, 5, 10)

Partial credit:

- Any work of merit such as correct or consistent numerator or denominator in fraction format.

Full credit:

- Correct answer without work shown.

(b)(ii) Scale 5C (0, 2, 4, 5)

Low partial credit:

- Any work of merit such as a correct or consistent partial probability e.g. $\frac{10}{52}$.

High partial credit:

- Almost correct or consistent response e.g. $\frac{13}{52}+\frac{9}{52}$ and stops.

Full credit:

- Correct answer without work shown.

(b)(iii) Scale 5C* (0, 2, 4, 5)

Low partial credit:

- Any work of merit such as a correct or consistent numerator or denominator in fraction format.

High partial credit:

- Almost correct or consistent response e.g. $\frac{30}{52}\times\frac{29}{51}$ and stops.

Full credit:

- Correct answer without work shown.

Question 3

(a) Scale 15C (0, 5, 10, 15)
Low partial credit:
- Any work of merit such as showing knowledge of plotting a point.

High partial credit:
- An almost correct response such as two or three points plotted correctly or a consistent error in plotting all four points.

(b) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit e.g. incomplete statement(s) with some merit.

(c) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as identification of a relevant formula.

High partial credit:
- Almost correct response such as one pair of lines shown as parallel or a correct response using a method not described in (b).

Question 4

(a) Scale 10C (0, 4, 7, 10)

Low partial credit:

- Any work of merit such as identification of a relevant formula.

High partial credit:

- An almost correct response e.g. mishandles r^2 .

(b) Scale 5C (0, 2, 4, 5)

Low partial credit:

- Any work of merit such as a correct substitution.

High partial credit:

- Almost correct response such as required conclusion not given.

(c) Scale 5C (0, 2, 4, 5)

Low partial credit:

- Any work of merit e.g. use of translation indicated.

High partial credit:

- Almost correct response such as correct centre and radius indicated.
- Correct answer without work shown.

(d) Scale 5C (0, 2, 4, 5)

Low partial credit:

- Any work of merit such as identification of a relevant formula.

High partial credit:

- An almost correct response such as slope of tangent found correctly but fails to finish.
- Correct answer without work shown.

Question 5A

(a)(i) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit such as an incomplete statement with radius or diameter or point of contact mentioned.

(a)(ii) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit e.g. any incorrect line through the point of contact.

(b) Scale 15D (0, 4, 7, 11, 15)
Low partial credit:
- Any work of merit such as an arc drawn from any vertex of the triangle.
- Midpoint of a triangle side indicated or finds the incentre or centroid.

Middle partial credit:
- Further work of merit such as one perpendicular bisector of the triangle side constructed correctly.

High partial credit:
- An almost correct response where the circumcentre is established correctly but no circle drawn or both mediators and circle drawn correctly but no evidence of construction.

Question 5B

Scale 25C (0, 8, 17, 25).
Low partial credit:
- Any one correct step.

High partial credit:
- One or two steps missing or incorrect.

Section B

Question 6

(a)(i) Scale 10C (0, 4, 7, 10)
Low partial credit:
- Any work of merit such as a rectangle for one year represented correctly or a correct scaled axis shown.

High partial credit:
- An almost correct response such as one or two incorrect rectangles or scales omitted.

(a)(ii) Scale 10C (0, 4, 7, 10).
Low partial credit:
- Any work of merit e.g. at least two correct elements identified.

High partial credit:
An almost correct response such as formula for mean with fully correct substitution.

Full credit:
- 121 128 without work.

(a)(iii) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit e.g. a correct relevant figure identified 86 932 or 54 432.

High partial credit:
- An almost correct response such as one step missing.

Full credit:
- Correct answer without work shown.

(a)(iv) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit which includes any mention of the year 2009, a decline in sales or an increase in sales.

(a)(v) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit which includes any mention of an increase of sales in year 2007, a decline in sales in years 2008/2009 or an increase in sales in years 2010/2011.

(b)(i) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit such as a correct relevant observation that is incomplete.

(b)(ii) Scale 5A (0, 5)
No credit:
- No reason given.

(b)(iii) Scale 5B (0, 2, 5)
Partial credit:
- Any work of merit such as an estimate within the range of values 72 000 to 90 000 without justification or any work with 36 081.

(c)(i) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as finding any or all of the correct percentages or fractions.

High partial credit:
- An almost correct response such as all angles indicated but drawn incorrectly or not drawn.
- Correct answer without work shown

(c)(ii) Scale 5B (0, 2, 5)
Partial credit:
- Selects B or C.

(d)(i) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit e.g. a correct element of stem-leaf plot indicated.

High partial credit:
- An almost complete response such as an incomplete plot with one/two elements missing or incorrect or no key indicated.

(d)(ii) Scale 5C (0, 2, 4, 5)
Low partial credit:
- Any work of merit such as an answer with no reference to stem-leaf or central tendency.

High partial credit:
- An almost complete response such as reference to median/mean but no conclusion.

(d)(iii) Scale 5C (0, 2, 4, 5).
Low partial credit:
- Any work of merit such as an answer with no reference to stem-leaf or measure of variability.

High partial credit:
- An almost complete response such as reference to range or measurement of variability but no conclusion.

Question 7

(a) Scale 5B (0, 2, 5).

Partial credit:

- Any work of merit such as any triangle drawn with no measurement of sides indicated or defines an isosceles triangle.

Full credit:

- A triangle drawn with three different lengths indicated.

(b) Scale 15D* (0, 4, 7, 11, 15).

Low partial credit:

- Any work of merit e.g. an indication that [*FG*] is an enlargement of [*BC*] or [*ED*] or work towards scale factor.

Middle partial credit:

- Further work of merit such as scale factor found.

High partial credit:

- An almost complete response such as scale factor applied to [*BC*] to get [*ED*] correctly or fails to finish e.g. leaves $|FG|$ as $1\cdot 25^2 \times 8$ or $1\cdot 25 \times 10$.

Full credit:

- Correct answer without work shown.

(c) Scale 15D* (0, 4, 7, 11, 15)

Low partial credit:

- Any work of merit such as identifying the cosine rule as a method of solution.

Middle partial credit:

- Further work of merit such as significant correct substitution into the cosine formula.

High partial credit:

- An almost complete response such as cosine rule worked to $\sqrt{73}$.
- Correct answer without work shown.

(d) Scale 5B* (0, 2, 5)

Partial credit:

- Any work of merit such as an effort at finding or explaining the centre of enlargement or an effort at the use of the scale factor to find the centre of enlargement.
- Correct answer without work shown.

(e) Scale 10D (0, 2, 5, 8, 10)

Low partial credit:

- Any work of merit e.g. vertical height indicated (from point *G* or in triangles *CBD, EDF*) or correct relevant formula identified or $|FG| = 12\cdot 5$.
- Correct answer without work shown.

Middle partial credit:

- Further work of merit such as α or sine α found.

High partial credit:

- An almost complete response whereby *h* is found but no conclusion is given.

Section C

Question 8

(a)(i) Scale 5B* (0, 2, 5)
Partial credit:
- Any work of merit e.g. leaves diameter = 4 or works with $l = \frac{16}{4}$.

Full credit:
- Correct answer without work shown.

(a)(ii) Scale 5B* (0, 2, 5)
Partial credit:
- Any work of merit such as identification of r in this part.

Full credit:
- Correct answer without work shown.

(b)(i) Scale 10C* (0, 4, 7, 10)
Low partial credit:
- Any work of merit such as one or two correct measurements shown.
- If (b) (i) not done but at least one correct measurement is shown in (b) (ii).

High partial credit:
- An almost correct response such as one or two incorrect measurements.
- If (b) (i) not done but all correct measurements are shown in (b) (ii).

(b)(ii) Scale 15D* (0, 4, 7, 11, 15)
Low partial credit:
- Any work of merit such as a correct substitution of h, F + L or TOFE.
- Correct answer without work shown.

Middle partial credit.
- Further work of merit such as two correct substitutions of h, F + L or TOFE.
- One correct substitution followed by some correct calculation.

High partial credit:
- An almost correct response such as fully correct substitution followed by an error in calculation.
- Two correct substitutions followed by fully correct calculation.

(c)(i) Scale 10D* (0, 2, 5, 8, 10)
Low partial credit:

- Any work of merit such as r identified as $1 \cdot 5$.

Middle partial credit:

- Further work of merit e.g. correct substitution for *r* and *h* or volume of cylinder or cone correct.

High partial credit:

- An almost correct response such as *h* isolated.

(c)(ii) Scale 5B* (0, 2, 5)
Partial credit:

- Any work of merit e.g. finds base or height of the box.

MARCANNA BREISE AS UCHT FREAGAIRT TRÍ GHAEILGE

(Bonus marks for answering through Irish)

Ba chóirmarcanna de réiranghnáthráta a bhronnadhariarrthóirínachngnóthaíonnníosmóná 75% d'iomlánnamarcanna don pháipéar.   Ba chóirfreisinan marc bónais sin a shlánú**síos**.

Déantarancinneadhagus an ríomhaireachtfaoin marc bónais i gcásgachpáipéirarleithligh.

Is é 5% angnáthrátaagus is é 300 iomlánnamarcanna don pháipéar. Mar sin, bainúsáid as an ngnáthráta 5% i gcásiarrthóirí a ghnóthaíonn 225 marc nóníoslú, e.g. 198 marc $\times$ 5% = 9·9 $\Rightarrow$bónas = 9 marc.

Mághnóthaíonn an t-iarrthóirníosmóná 225 marc, ríomhtar an bónas de réirnafoirmle
[300 – bunmharc] $\times$15%, agusan marc bónais sin a shlánú**síos**. In ionadanríomhaireacht sin a dhéanamh, is féidirúsáid a bhaint as an táblathíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0